

# The Shoenberg Effect in Relativistic Degenerate Electron Gas and Observational Evidences in Magnetars

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## ABSTRACT

In the neutron star, there is the relativistic degenerate electron gas which is the densest in the Universe. Due to electron-electron magnetic interaction, the differential susceptibility can equal or exceed 1, which results in the magnetic system of a neutron star being metastable or unstable states. The Fermi liquid of nucleons under the crust can be in the metastable state, while the crust is in the unstable state, which results in a phase transition occurring and the formation of domain structures in the later. The magnetizations of adjacent magnetic domains have opposite directions. The shearing stress acting on adjacent domains can result in a series of shifts or fractures in the crust. The crust releases the magnetic free energy, which corresponds to the bursts observed in magnetars. Simultaneously, a series of shifts or fractures which the deep crust closed to the Fermi liquid of nucleons undergoes can trigger the phase transition of the Fermi liquid of nucleons from the metastable state to a stable state. The free magnetic energy in the Fermi liquid of nucleons is released, which corresponds to the giant flares observed in some magnetars.

*Subject headings:* star:neutron—magnetic fields—Pulsar: general

## 1. Introduction

In 1930, de Haas and van Alphen first observed that the magnetization  $\vec{M}$  in normal metals oscillates, under applied intense magnetic field and low temperature. The oscillatory functions are sinusoidal series with its fundamental frequency which can be described by the extremal areas of the cross-section of the Fermi surface normal to the applied magnetic field (Lifshits & Kosevich 1956). Using the impulsive field method, Shoenberg found an unexpectedly high amplitude for the second harmonic and proposed magnetic interaction between the conduction electrons (Shoenberg 1962), which was so called Shoenberg effect. He suggested that the magnetizing field is not applied field  $\vec{H}$  but magnetic induction  $\vec{B}$ . When the differential magnetic susceptibility  $\chi_m = \partial(4\pi M)/\partial B$  exceeds 1 (We use the Gaussian units in the paper), the spatially

uniform state of electron gas is thermodynamically unstable. Then, the electron gas rapidly evolves into a stable state, and has a spatially inhomogeneous magnetic field with domains which are called as Condon domains (Condon 1966).

The magnetization and the de Haas-van Alphen oscillating effect for a relativistic degenerate electron gas have been studied by many authors (Visvanathan 1962; Canuto & Chiu 1968; Chudnovsky 1981). Previous studies about the magnetic susceptibility of neutron stars focused on whether the observed field results from a spontaneous magnetization (Lee et al. 1969; O'Connell & Roussel 1971). This almost cannot occur because the neutron star is insufficiently cool. The domain structure and the potential applications in neutron star's crusts have been postulated by Blandford and Wilkes (Blandford & Hernquist 1982; Wilkes & Ingraham 1989). The inhomogeneous field resulting from domain formation in the crust of a normal neutron star is negligible and can not produce observable

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effects. However, for magnetars with magnetic fields stronger than normal neutron stars, the inhomogeneous field is significant so that magnetars can produce super-Eddington X-ray outbursts.

Magnetars including Anomalous X-ray Pulsars (AXPs) and Soft Gamma Repeaters (SGRs) are characterized by their inferred dipolar magnetic field strength ranging from  $5.9 \times 10^{13}$  G to  $1.8 \times 10^{15}$  G and their long spin periods ranging from 5.2 s to 11.8 s. Surprisingly, some magnetars have undergone giant flares (GFs) in which the energy up to  $10^{46}$  ergs is released in a fraction of a second via  $\gamma$ -ray emissions. Up to now, three GFs have been observed from respectively SGR 0526-66 (Cline et al. 1982), SGR 1900+14 (Mazets et al. 1999) and SGR 1806-20 (Palmer et al. 2005).

Their X-ray and  $\gamma$ -ray luminosities during persistent and burst phase are too large to be powered by their kinetic energy. Where is the magnetic energy released stored prior to the GFs? Is it in the magnetospheres or in the neutron star? Duncan & Thompson (1992) and Thompson & Duncan (2001) proposed a popular model for magnetars, and they considered that the released magnetic energy is stored in the neutron star. The controversy for the popular model is about its triggering mechanism for high energy radiation. Duncan & Thompson (1992) suggested that a helical distortion of the magnetic field in the core induces a large-scale fracture in the crust, and a twisting deformation of the magnetic field in the crust and the magnetospheres. A GF may involve a large disturbance which probably is driven by a rearrangement of magnetic field in the deep crust and core (Thompson & Duncan 1995), while the persistent emission can be explained by the very slow transport of the field from the core to the crust by Hall drift (Goldreich & Reisenegger 1992). Kondratyev (2002) first postulated the existence of magnetic domains in the magnetar's crust because of the inhomogeneous crust structure. He argued that the burst activity of SGRs could originate from magnetic avalanches. However, because of the domain forming mechanism resulting from the interaction between nucleus spins, the required magnetic field is in the range  $10^{16} \sim 10^{17}$  G. Whether the magnetic field in the crust of neutron star can be such strong is still controversial. Having considered the GF's emission energy ( $10^{42} - 10^{46}$  ergs) and its mean

waiting time (Mazets et al. 1979; Hurley et al. 1999; Terasawa et al. 2005), Stella et al. (2005) estimated that the internal field strength of magnetar at birth time can reach up to  $B \geq 10^{15.7}$  G. Later, having numerically calculated the magnetic properties of magnetar-matter such as the magnetization and the susceptibility of electron, proton and neutron, Suh & Mathews (2010) suggested a magnetic domain model to correlate smoothly between the statistics of star-quakes and magnetic avalanches in magnetar crusts.

The phase transition to the domain phase in magnetars is different from the ferromagnetic phase transition. The magnetic ordering in iron comes from the exchange interaction of bound electron spins at a sufficient low temperature, and it is not prerequisite to have an external magnetic field. While the magnetic ordering in magnetar arises from the interaction between the orbital magnetic moments of free electrons under high-quantizing field. Therefore, given a nearly uniform distribution of electrons, the phase transitions in magnetars are similar to those in beryllium where magnetized matter is the conductive electrons (Shoenberg 1962). The relativistic Fermi sea of electrons is only slightly perturbed by the Coulomb forces of nuclei in the crust of magnetar and does not efficiently screen nuclear charges. The dominant contribution to differential susceptibility in a magnetar comes from electrons, while the contributions from nucleons are negligible (the magnetic moments of nucleons are three orders lower than electron orbital moments).

In this paper we analytically calculate the differential susceptibility of relativistic degenerate electron gas, and find it is an oscillated function of magnetic field which is called as the de Haas and van Alphen oscillation. In §2, we discuss the magnetic phase transition. The differential susceptibility is calculated in §3. In §4, we use our model to explain the observed evidences in magnetars. Summary is in §5.

## 2. The magnetic phase transition

The magnetization of degenerate electron gas at high-quantizing magnetic field and low temperature ( $KT \ll \hbar\omega_c$ ) should exhibit a nonlinear de Haas-van Alphen effect. Without the magnetic interaction the oscillated magnetization are assumed

to be periodical in magnetic field  $H$  rather than in  $1/H$  (Pippard 1980; Shoenberg 1984):

$$\tilde{M} = M_0 \sin(H/H_0), \quad (1)$$

where  $H_0$  hardly varies over one cycle of oscillation. However, if we consider the feedback contribution from  $4\pi\tilde{M}$ , the magnetization depends on not only the quantizing magnetic field but also the cooperating ordering of the magnetic moments themselves. The magnetization actually should be a function of magnetic induction,  $\tilde{M} = \tilde{M}(\vec{B})$ . Then, the magnetic induction is given by  $\vec{B} = \vec{H} + 4\pi\tilde{M}(\vec{B})$ . Replacing  $\vec{H}$  by  $\vec{B}$  is called as the Shoenberg or  $\vec{H}$ - $\vec{B}$  effect. The oscillated sinusoidal function of the magnetization on magnetic field in Eq.(1) is given by

$$\tilde{M} = M_0 \sin(B/H_0) = M_0 \sin[(H + 4\pi M)/H_0], \quad (2)$$

where  $M_0$  and  $H_0$  are constants in a period.

When  $4\pi M_0/H_0 > 1$ , as Fig.1 shows,  $\tilde{M}$  becomes a three-values function of  $H$ , and the differential magnetic susceptibility  $\chi_m = \partial(4\pi\tilde{M})/\partial B$  can exceed unity. Having considered the case of a thin rod oriented along the direction of magnetization, Pippard (1963) showed that the multi valued function in Fig.1 is not physical. In the region of the curve between L and L' where the slope is lower than unity, the magnetized states are unstable and can never really exist. For any weak perturbation  $\delta\vec{H}$  near  $\vec{H}_0$ , the perturbation of magnetic induction  $\delta\vec{B}$  is given by

$$\delta\vec{B} = \frac{\delta\vec{H}}{1 - \chi_m}. \quad (3)$$

Obviously, there is a singular point when  $\chi_m = 1$  in Eq.(3), where the magnetized state is unstable and the first-order phase transition should occur. Then, the magnetic system should be in a stable state in which magnetization is inhomogeneous and magnetic domains form. The magnetizations in the adjacent domains have opposite directions. The stable magnetized states are represented by the dashed line between N and N' in Fig.1. But if there is a surface energy at the boundary of two different magnetization it may become metastable, which is similar to superheating or supercooling in gas-liquid transition (Reichl 1998). The solid line between L and N or L' and N' in Fig.1 represents the metastable state. It is not difficult

to achieve the condition of the above magnetic phase transition in the terrestrial lab. The experiments of Condon indicated that the magnetized phase transition can take place in some metals (Condon 1966). However, what is the situation in compact objects such as neutron star? In the following we calculate the differential susceptibility of relativistic degenerate electron gas and show the observable effects of magnetic phase transition in neutron star.

### 3. The differential susceptibility of relativistic degenerate electron gas

The assembly of electrons in neutron star under a strong magnetic field is degenerated and relativistic. The energy eigenvalues are (Johnson & Lippmann 1949)

$$E = [c^2 p_z^2 + \mu^2 + \mu\epsilon_c(2n + s + 1)]^{1/2}, \quad (4)$$

where  $\mu = m_e c^2$  and  $\epsilon_c = \hbar e B / c m_e = \hbar \omega_c$  are the rest energy and cyclotron energy of electron, respectively. Here,  $p_z$  is the momentum component along field direction which is the z-direction,  $n = 0, 1, \dots$  and  $s = \pm 1$  are the Landau and spin quantum number, respectively. The density of states per unit volume for the energy level is given by  $(p_z/h)(eB/hc) = g(p_z/h)$  where  $g = eB/hc$  is the density of states per unit area for a Landau energy level. As a consequence of quantization, the spherical Fermi surface of the free electron is replaced by a set of circles located on a spherical surface with a common axis along the  $B$  direction, which is shown in Fig.2.

The susceptibility of the electron assembly can be obtained by finding its grand potential which depends on the density of states. It is a function of energy and magnetic induction. The density of states per unit volume is given by

$$Z(\epsilon, B) = \frac{2eB}{c^2 \hbar^2} \sum_{n,s} [\epsilon^2 + 2\epsilon\mu - \mu\epsilon_c(2n + s + 1)]^{1/2}, \quad (5)$$

where  $\epsilon = E - \mu$  is the kinetic energy of an electron. The grand potential per unit volume of the assembly is given by

$$\begin{aligned} J &= -\beta \int \ln[1 + e^{\beta(\psi - \epsilon)}] dZ(\epsilon, B) \\ &= -\int_0^\infty Z(\epsilon, B) f(\epsilon) d\epsilon, \end{aligned} \quad (6)$$

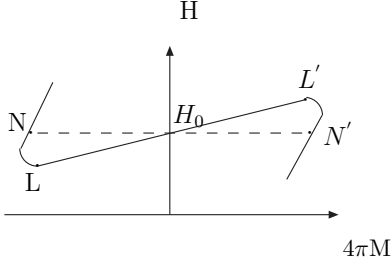


Fig. 1.— Magnetic field  $H$  vs. magnetization  $4\pi\tilde{M}$

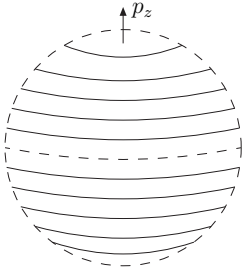


Fig. 2.— The free-electron Fermi surface in the presence of a magnetic field along the  $p_z$ -axis.

where  $\beta = (kT)^{-1}$ ,  $\psi$  is the chemical potential,  $f(\epsilon)$  is the Fermi-Dirac distribution function. After summing on spin quantum number, Eq.(5) reduces to

$$Z = \frac{2}{\sqrt{\pi}} g^{3/2} [b^{1/2} + 2 \sum_{n=1}^{[b]} (b-n)^{1/2}], \quad (7)$$

where

$$b = \frac{\epsilon^2 + 2\epsilon\mu}{2\epsilon_c\mu}, \quad (8)$$

and  $[b]$  is the integer of  $b$  ( $[b] \leq b$ ). For a neutron star, the electron system is almost completely degenerate ( $\psi \gg \beta^{-1}$ ) and the Fermi-Dirac distribution function is almost a step function except the region near  $\epsilon = \psi$ . The magnetic moment depends on the first-order derivative of grand potential with  $B$  and the dominant contributions come from the integral of Eq.(6) at the region near  $\epsilon = \psi$ . Hence, we define that  $b_m = b|_{\epsilon=\psi}$ . If the chemical potential of neutron star is  $\sim 10$  MeV and the magnetic field is the quantum field  $B_Q = 4.414 \times 10^{13} \text{G}$  (the cyclotron energy of electron equals to its rest energy), then  $[b_m]$  is about 200. Based on Eq.(7), we can see that the first-order derivative of grand potential with  $B$  has the singularity when  $b_m = [b_m]$ . Therefore, the susceptibility of relativistic degenerate electron gas can be equal to or even larger than 1.

For revealing the oscillating effect of the grand potential, we calculate the sum of Eq.(7) with the Poisson summation formula (Dingle 1952):

$$\frac{1}{2}F(0) + \sum_{n=1}^{\infty} F(n) = \sum_{r=-\infty}^{\infty} \int_0^{[b]} F(x) e^{i2\pi r x} dx. \quad (9)$$

Because  $[b]$  is much larger than 1, the density of states in Eq.(5) is approximately by

$$Z = \frac{4}{\sqrt{\pi}} g^{3/2} \left[ \frac{2}{3} b^{3/2} + \sum_{\nu=0,2,4}^{\infty} \frac{(-1)^{\nu/2} (2\nu-1)! B_{(\nu+2)/2}}{\sqrt{b} (4b)^{\nu} (\nu-1)! (\nu+2)!} + \frac{1}{2\sqrt{2\pi}} \sum_{r=1}^{\infty} \frac{\cos(2\pi r b - 3\pi/4)}{r^{3/2}} \right], \quad (10)$$

where  $B_m$  denotes the Bernoulli numbers. The first two terms on the right-hand-side are the non-oscillating parts while the third term is the oscillating part.

The non-oscillating grand potential per unit volume can be obtained from Eq.(10). As the first-order approximation, we can keep only the first

term in the summation and obtain

$$\bar{J} = -\frac{4}{3\sqrt{\pi}(2\mu\epsilon_c)^{3/2}}g^{3/2}\int_0^\infty(\epsilon^2+2\epsilon\mu)^{3/2}f(\epsilon)d\epsilon + \frac{\sqrt{2\mu\epsilon_c}}{6\sqrt{\pi}}g^{3/2}\int_0^\infty\frac{f(\epsilon)d\epsilon}{(\epsilon^2+2\epsilon\mu)^{1/2}}. \quad (11)$$

In general,  $\psi \gg \mu$  for neutron star. Using the standard methods of evaluating integrals for the Fermi-Dirac distribution, we can evaluate the integrals in Eq.(11). The non-oscillating susceptibility is given by

$$\bar{\chi}_m = 4\pi\frac{\partial^2\bar{J}}{\partial B^2} = \frac{4\sqrt{2\pi}}{3}\frac{\sqrt{\mu\epsilon_c}}{B^2}g^{3/2}[\ln\frac{2\psi}{\mu} - \frac{\pi^2}{6}(\frac{kT}{\psi})^2], \quad (12)$$

In our work, the chemical potential and the magnetic field in the deep crust of neutron star is about 10 MeV and  $10^{12}$  G, the non-oscillating susceptibility can be  $\sim 5.6 \times 10^{-3}$ .

Using a approximate formula for the Fermi-Dirac distribution

$$\int_0^\infty \eta(\epsilon)f(\epsilon)d\epsilon = \int_0^{\psi-\mu} \eta(\epsilon)d\epsilon + \frac{\pi^2}{6}(kT)^2\eta'(\psi-\mu)d\epsilon, \quad (13)$$

and the Fresnel Integrals

$$\int_0^x \cos(\frac{\pi}{2}t^2)dt \approx x \quad (x \ll 1) \\ \approx \frac{1}{2} + \frac{1}{\pi x} \sin(\frac{\pi}{2}x^2) \quad (x \gg 1), \quad (14)$$

we can obtain the oscillating term of the grand potential per unit volume as

$$\tilde{J} = \frac{\sqrt{2}}{\pi^{3/2}}g^{3/2}\sum_{r=1}^\infty\frac{1}{r^{3/2}}[(\frac{1}{2}\sqrt{\frac{\pi}{2a_r}}-\mu)\cos(a_r\mu^2+\frac{3}{4}\pi)+\frac{1}{2}\sqrt{\frac{\pi}{2a_r}}\sin(a_r\mu^2+\frac{3}{4}\pi)]+\frac{\sqrt{2}}{\pi^{3/2}}g^{3/2}\sum_{r=1}^\infty\frac{1}{r^{3/2}}\cdot\frac{1}{a_r\psi}[\frac{\pi}{2}-\frac{\pi^2}{3}(kT)^2(a_r\psi)^2]\sin(a_r\psi^2-a_r\mu^2-\frac{3}{4}\pi)], \quad (15)$$

where  $a_r = \pi r/(\epsilon_c\mu)$ . On calculating the differential susceptibility which is the second derivative of Eq.(15), we only keep the most rapidly varying terms, that is, only differentiate cosines and sinusoidal. Because  $\psi \gg \mu$  for neutron star, the last term in Eq.(15) is dominant. The oscillating susceptibility is approximately given by

$$\tilde{\chi}_m = A_0\sum_{r=1}^\infty(r^{-1/2}-A_1(\frac{kT}{\epsilon_c})^2r^{3/2})\cos(a_r\psi^2-\frac{3}{4}\pi), \quad (16)$$

where

$$A_0 = \alpha(\frac{\psi}{\mu})^3(\frac{B}{B_Q})^{-3/2}, \quad (17)$$

and

$$A_1 = \frac{2\pi^3}{3}(\frac{\psi}{\mu})^2. \quad (18)$$

Here,  $\alpha$  in Eq.(17) is the fine structure constant. The result of Eq.(16) is similar with the result in Visvanathan (1962). It indicates that the susceptibility oscillates with  $1/B$ . We replace  $B$  by  $B' = B - B_0$ . When  $B \rightarrow B_0$  we have  $1/B \approx 1/B_0(1 - B'/B_0)$ . Note that the first term is a constant, thus the oscillation are the period functions of  $B'$ . The differences of magnetic induction in an oscillating period are the periods of  $B'$ , which are presented by

$$\delta B = \frac{2\hbar ec^2 B_0^2}{\psi^2 r} \quad (r = 1, 2, \dots). \quad (19)$$

The coefficients of the cosine functions in Eq.(16) include two terms. The first term is the result of complete degeneracy, while the second term comes from the thermal fluctuation and is proportional to the second power of temperature. For a typical neutron star, chemical potential, magnetic field and temperature are the order of 10 MeV,  $10^{12}$  G, and  $10^6$  K, respectively. The second term can be ignored because of  $KT \ll \hbar\omega_c \ll \psi_0$ . The oscillating susceptibility,  $\tilde{\chi}_m$ , is mainly determined by  $A_0$ . Fig.3 shows the relation between  $\log A_0$  and  $B/B_Q$ . Obviously, the differential susceptibility can equal or exceed 1 when  $B < \sim 15B_Q$ . In this work, we assume that magnetars are similar to normal neutron stars except magnetic field: for a normal neutron star,  $B = 10^{12}$  G while  $B = 15B_Q$  for magnetars. For a stronger magnetic field, the condition  $\hbar\omega_c \ll \psi_0$  or  $[b_m] \gg 1$  cannot be satisfied, the above approximate methods cannot be used. We will discuss them in the next paper.

#### 4. The observed evidences in magnetars

As mentioned in the last section, the susceptibility of relativistic degenerate electron gas can be equal to or exceed 1. The electron gas under strong magnetic field in neutron star may be in unstable state, and the first-order phase transition should occur. Finally, the electron gas should be a stable state and has the Condon's domain structure. The magnetizations in the adjacent domains have opposite direction.

Based on Eq.(19), the relative variation of magnetic induction between adjacent magnetic do-

mains is

$$\frac{\delta B}{B_0} \sim 10^{-3} \frac{B_0}{B_Q}. \quad (20)$$

For a normal neutron star ( $B_0 \sim 10^{12}$  G),  $\delta B/B_0 \sim 10^{-4}$ , it is very difficult to observe the oscillatory effects. However, for magnetars ( $B_0 \sim 15B_Q$ ),  $\delta B/B_0 \sim 10^{-2}$ , which is large enough to produce the bursts in SGRs or AXPs.

The adjacent magnetic domains have different densities of electrons. The different magnetizations in the adjacent magnetic domains mean that the phase difference is  $\pi$ . According to Eq.(16), we have  $\delta(a_r\psi^2) = \pi$ . For simplicity we approximate the chemical potential  $\psi$  with zero temperature free-field Fermi energy  $\psi_0 = ch(3n/8\pi)^{1/3}$ . The difference of electron density between the adjacent magnetic domains is given by

$$\frac{\delta n}{n} \sim 10^{-2} \left(\frac{\epsilon_e}{20}\right)^{-2} \left(\frac{B_0}{15B_Q}\right), \quad (21)$$

where  $\epsilon_e = \psi_0/\mu$  and we have taken fixed  $B = B_0$ . For magnetar,  $\psi_0 \sim 10$  MeV,  $B_0 \sim 15B_Q$ , and the rest energy of electron  $\mu \sim 0.5$  MeV. Obviously, here,  $\epsilon_e$  is scaled to 20 and  $B_0$  to  $15B_Q$ , respectively.

The electrons in a neutron star coexist with other components such as nuclei in the crust or Fermi liquid of protons and neutrons closed to the deep crust. Due to the Coulomb interactions between electrons and nuclei or protons, the matter of neutron star may not be homogeneous once the domain structure appears in the crust. However, it can not occur in the Fermi liquid. The mechanism of forming electron domain is the interactions between orbital magnetic moments which are counterbalanced by the degenerate pressure of electrons  $P_e$ . Because the Coulomb interaction of the crystal lattices in the crust is the same order of magnitude with the interaction between orbital magnetic moments, the electron domain structure can lead to the forming of the nuclei domain structure. However, in Fermi liquid of nucleons, the neutron degenerate pressure  $P_n \gg P_e = Y_e P_n$ , where  $Y_e$  is the fraction of electrons per neutron and it is only several tenths. The interaction between orbital magnetic moments can not counterbalance against the neutron degenerate pressure  $P_n$ . Therefore, the magnetic domain structure cannot form in Fermi liquid of nucleons. The magnetic system is in a metastable state, which is

similar to the supercooling or superheating states in the first-order phase transition.

In the crust of magnetar we can roughly calculate the size of the domain structure. With the density increasing, the quantum number of Landau energy also increases. Using the gravitational potential energy of a nucleon and the Fermi energy per electron, we can estimate the height of magnetic domains by (Suh & Mathews 2010)

$$\delta z \sim 10^3 \left(\frac{g}{10^{14}}\right)^{-1} \left(\frac{Y_e}{0.36}\right) \left(\frac{\epsilon_e}{20}\right)^{-1} \left(\frac{B_0}{15B_Q}\right) \quad \text{cm}, \quad (22)$$

where  $g$  is the surface gravity and  $Y_e$  is scaled to 0.36 when neutrons drip out from nuclei. If we assume that the volume change of the electron gas during forming domains is only perpendicular to the radial direction of neutron star and the changing magnitude (or the thickness of domain walls) is  $\sim$  a cyclonic radius of electron, based on Eq.(21), the width of magnetic domain can be estimated by

$$\delta l \sim 10^{-9} \left(\frac{\epsilon_e}{20}\right)^3 \left(\frac{B_0}{15B_Q}\right)^{-2} \quad \text{cm}. \quad (23)$$

The Maxwell shearing stress between the adjacent domains can deform the crust and result a strain,  $\theta$ . It is given by

$$\frac{B_0 \delta B}{4\pi} \sim \theta \nu \sim \frac{\theta B_\nu^2}{4\pi}, \quad (24)$$

where  $\nu$  is the shear modulus of the crust and  $B_\nu = \sqrt{4\pi\nu}$ . Closed to the bottom of the crust,  $B_\nu \simeq 6 \times 10^{15}$  G (Baym & Pines 1971). For magnetars,  $B_0 \sim 15B_Q$ , according to Eqs.(20) and (24), we have the strain  $\theta \sim 10^{-3}$ . Ruderman (1991) calculated that the maximum strain of the crust,  $\theta_{\max}$ , is  $\sim 10^{-2} - 10^{-4}$ . Our result is in its range.

When the domain structures appear in the crust of magnetar, the shearing stress acting on adjacent domains can produce a relative shift or the fracture between them. This sudden shift or fracture can propagate with Alfvén velocity along the domain layers, which results in a series of shifts or fractures in magnetic domains. The time scale of shifts propagating along a domain layer is estimated by

$$\tau \sim \frac{l}{V_A} \sim 0.1 \left(\frac{B_0}{15B_Q}\right)^{-1} \left(\frac{\rho}{10^{15}}\right) \left(\frac{l}{R_*}\right) \quad \text{s}, \quad (25)$$

where  $V_A$  and  $\rho$  are the Alfvén velocity (Duncan 2004) and the matter density at the deep crust, respectively,  $l$  is the length of domain layer and  $R_*$  ( $\sim 10\text{km}$ ) is the radius of magnetar. The time scale is agree with the observed duration of SGRs bursts (Thompson & Duncan 1995). The available magnetic free energy is then

$$E_{\text{burst}} \sim 10^{41} \left( \frac{\delta B}{10^{13}} \right)^2 \left( \frac{\delta z}{10^3} \right) \quad \text{ergs}, \quad (26)$$

where we scaled  $\delta z$  to  $10^3$  cm (See Eq. (22)) and the  $\delta B$  to  $10^{13}$  G (See Eq.(20)). This energy also agree with the observation of bursts.

The Fermi liquid of nucleons in magnetar may be in metastable state. A series of shifts which the deep crust closed to the Fermi liquid of nucleons undergoes can trigger the phase transition of the Fermi liquid of nucleons from the metastable state to a stable state. The free magnetic energy in the Fermi liquid of nucleons is released and it is far greater than that in the crust because the magnetic induction and the thickness of the Fermi liquid of nucleons is larger than the crust. Let the magnetic induction of Fermi liquid of nucleons be at the same order of magnitude with that in deep crust, and the thickness of Fermi liquid of nucleons be approximated by the radius of magnetar. We estimate that the energy released is

$$E_{\text{flare}} \sim 10^{44} \left( \frac{\delta B}{10^{13}} \right)^2 \quad \text{ergs}, \quad (27)$$

This energy is agree with those released in GFs of some magnetars.

## 5. Summary

We discussed the magnetization effects of relativistic degenerate electron gas in neutron star. Having considered the magnetic interaction between electrons, we found that the magnetic systems may be unstable or metastable when the differential susceptibility equals or exceeds 1. Under an ultra-strong magnetic field, the magnetic domain structures in magnetar can appear in the solid crust, while Fermi liquid of nucleons may be in metastable state. The shearing stress acting on adjacent domains can result in a series of shifts or fractures in the crust. The crust releases the magnetic free energy, which corresponds to the bursts

observed in magnetars. Simultaneously, a series of shifts or fractures which the deep crust closed to the Fermi liquid of nucleons undergoes can trigger the phase transition of the Fermi liquid of nucleons from the metastable state to a stable state. The free magnetic energy in the Fermi liquid of nucleons is released, which corresponds to the GFs observed in some magnetars.

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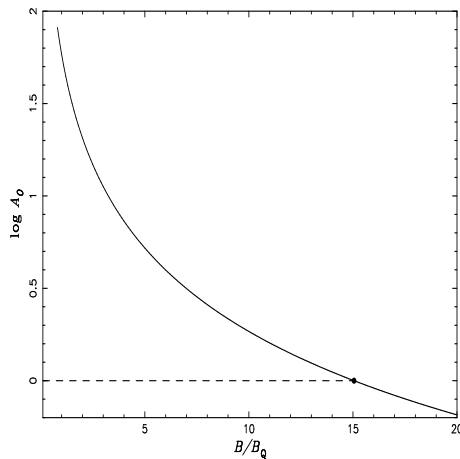


Fig. 3.—  $\log A_0$  vs.  $B/B_Q$